 Notes 3.4

3.4 The Graph of a Rational Function; Inverse and Joint Variation

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Intercepts (Section 1.2, pp. 15–17)
- Even and Odd Functions (Section 2.3, pp. 80–82)

Now work the 'Are You Prepared?' problems on page 207.

OBJECTIVES
1. Analyze the Graph of a Rational Function
2. Solve Applied Problems Involving Rational Functions
3. Construct a Model Using Inverse Variation
4. Construct a Model Using Joint or Combined Variation

1 Analyze the Graph of a Rational Function

Graphing utilities make the task of graphing rational functions less time consuming. However, the results of algebraic analysis must be taken into account before drawing conclusions based on the graph provided by the utility. We will use the information collected in the last section in conjunction with the graphing utility to analyze the graph of a rational function $R(x) = \frac{p(x)}{q(x)}$. The analysis will require the following steps:

Analyzing the Graph of a Rational Function

Step 1: Find the domain of the rational function.

Step 2: Write R in lowest terms.

Step 3: Locate the intercepts of the graph. The x-intercepts, if any, of $R(x) = \frac{p(x)}{q(x)}$ in lowest terms, satisfy the equation $p(x) = 0$. The y-intercept, if there is one, is $R(0)$.

Step 4: Test for symmetry. Replace x by $-x$ in $R(x)$. If $R(-x) = R(x)$, there is symmetry with respect to the y-axis; if $R(-x) = -R(x)$, there is symmetry with respect to the origin.

Step 5: Locate the vertical asymptotes. The vertical asymptotes, if any, of $R(x) = \frac{p(x)}{q(x)}$ in lowest terms are found by identifying the real zeros of $q(x)$. Each zero of the denominator gives rise to a vertical asymptote.

Step 6: Locate the horizontal or oblique asymptotes, if any, using the procedure given in Section 3.3. Determine points, if any, at which the graph of R intersects these asymptotes.

Step 7: Graph R using a graphing utility.

Step 8: Use the results obtained in Steps 1 through 7 to graph R by hand.

EXAMPLE 1 **Analyzing the Graph of a Rational Function**

Analyze the graph of the rational function: $R(x) = \frac{x - 1}{x^2 - 4}$

Ex.1 → Analyze the graph of $R(x) = \frac{x-1}{x^2-4}$

— Factor numerator & denominator → $\frac{x-1}{(x+2)(x-2)}$

① Find the domain (note: Do not cancel common factors yet, if there are any)

- Set denominator equal to zero & solve. Any real solutions will be your domain restriction. $D: \{x \mid x \neq -2, 2\}$

② Write R in lowest terms (note: now is when you would cancel any common factors & rewrite function w/ cancelled factors removed)

③ X-intercept : Set numerator equal to zero + solve.

$x - 1 = 0 \rightarrow x = 1 \rightarrow \boxed{(1,0)}$

y-intercept : Plug zero in for x + solve. $y = \frac{(0)-1}{(0)^2-4} = \frac{1}{4}$

$\boxed{(0, \frac{1}{4})}$

④ Test for symmetry : • If $R(-x) = R(x)$ → sym. w/resp. to y-axis
• If $R(-x) = -R(x)$ → sym. w/resp to origin

$R(-x) = \frac{(-x)-1}{(-x)^2-4} = \frac{-x-1}{x^2-4} = \frac{-(x+1)}{x^2-4}$ → neither even or odd, so $\boxed{\text{No symmetry}}$

⑤ Vertical Asymptotes : (After all common factors have been cancelled, set denominator equal to zero + solve)

$\frac{x-1}{(x+2)(x-2)} \longrightarrow \boxed{x = -2,\ x = 2}$ → VA's

⑥ Horizontal or Oblique Asymptotes : since $n < m$, $\boxed{\text{HA is } y = 0}$

⑦ Graph R using graphing calc.

⑧ Use results in steps 1-7 to graph R by hand →